**EE 505**
**Experiment 3**
**Spring 2025**

## Statistical Analysis and Yield Prediction in Data Converters

In the previous experiment the yield of an R-2R DAC was investigated. Discrete thin film resistors were used to facilitate experimental validation in Laboratory Experiment 1. Physical resolution was limited to 4 bits though yield at the 10-bit level was assessed. The only reason we kept the physical resolution at the 4-bit level was to facilitate reduced computational complexity. With realistic constraints, the results were indicative of what would be attainable with higher resolution.

In this experiment, the comparison of two of the most popular resistive DAC structures will be compared. One will be the R-2R DAC and the other the string DAC. Again, we will limit physical resolution to 4 bits and target yield assessment at the 10-bit level. Hopefully the results you obtained from the previous experiment can be used as part of the assessment of this experiment.

One of the major factors in DAC design is the amount of area allocated to the matching critical components. To fairly compare two different architectures, the area allocated to the matching critical components should be the same. In Laboratory Experiment 1, discrete thin-film resistors were used and neither the characteristics of the thin films nor the area of the thin films were considered. So lets assume the discrete resistors you have been using come from a process where the area of the resistors is $100\mu m^2$. Thus you can assume that the 4-bit R-2R DAC you designed, comprised of 13 resistors, had a total resistor area of 1300 $\mu m^2$.

### Part 1

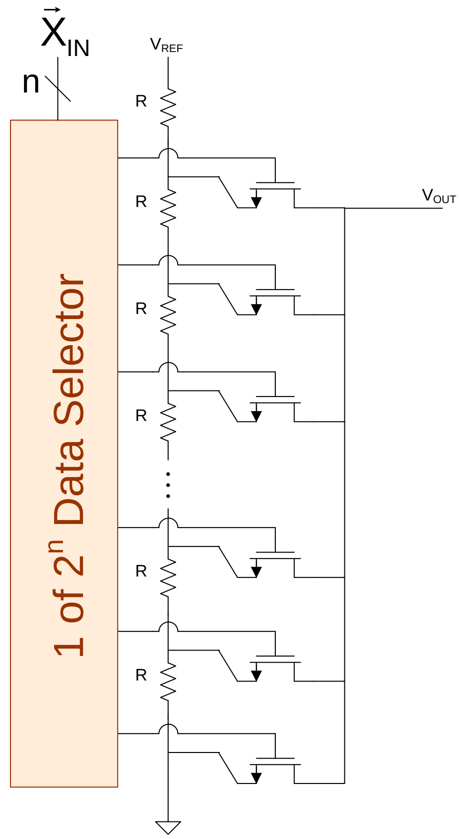
Determine the equivalent Pelgrom parameter for the resistors you measured in Laboratory Experiment 1.

A string DAC is shown in the following figure. The DAC output is obtained by closing one of the MOS switches to present the corresponding TAP voltage at the output. Assume that the switches are ideal.

### Part 2

Design this string DAC using the same total area as was used for the R-2R DAC (1300 $\mu m^{2)}$ . Determine the yield assuming an specification of $ENOB_{INL}$=10bits. Assume the switches are ideal.

In the previous R-2R DAC design, the area allocated to the shunt resistors was twice the area allocated to the series resistors. Alternatively, the unit resistor could have been used for the "2R" resistor and two of these unit resistors could have been placed in parallel to form the "R" resistor. We will term the previous R-2R DAC the "Series DAC" and the modified DAC the "Parallel DAC"




## Part 3

Design the R-2R DAC using the "Series DAC" structure using the same total area that was used for Part 2. Determine the yield assuming the same INL specification of $ENOB_{INL}$=10Bits. Assume the switches are ideal.

## Part 4

Design the R-2R DAC using the "Parallel DAC" structure using the same total area that was used for Part 1. Determine the yield assuming the same INL specification of $ENOB_{INL}$=10Bits. Assume the switches are ideal.

## Part 5

Compare the yield of the three different DAC structures considered which all have the same total resistor area.

Though the INL is invariably of interest in many DAC structures, accurate analytical expressions for the INL for data converters with over 3 bits of resolution are not mathematically tractable so in this experiment we will also consider the mid-code $INL_k$. which is analytically tractable. By knowing the relationship between the mid-code $INL_k$ and the actual INL for a given architecture, analytical estimates of the INL is possible.

When doing statistical calculations in MATLAB or SPECTRE, keep track of your seed for the random number generator. If you use the same seed each time you will get the same results. This will allow you to go back and repeat a particular simulation if necessary.

## Part 6

a) Obtain an analytical expression for the mid-code $INL_k$ defined as $INL_{\frac{N}{2}}$ for an n-bit string DAC and from this expression, determine the mean and standard deviation of $INL_{\frac{N}{2}}$

b) Using MATLAB or SPECTRE, simulate a large number of DACs and determine the mean and standard deviation of $INL_{\frac{N}{2}}$ for n=10. Use a Poly resistor in whatever process you are most familiar with and assume the area of each resistor is $10\mu m^2$. If you use MATLAB, assume the resistor population is a modified Gaussian[1] distribution and use the Pelgrom parameter from a resistor in the process you are most familiar with for characterizing the resistor in MATLAB. Compare these results to those obtained analytically in part a). You should do a large enough number of simulations to validate that MATLAB or SPECTRE provides results that approach the theoretical results.

c) Determine which tap position (from 0 to 1023) will have the largest standard deviation in $INL_k$

d) Determine the mean and standard deviation of the INL for this 10-bit string DAC using either a MATLAB statistical analysis or Spectre Monte-Carlo simulations. Also, obtain an approximate distribution of the INL for this DAC and compare with that of the mid-code $INL_k$. (Is the INL a zero-mean random variable? Is the INL Gaussian? Is it modified Gaussian'?)

e) If the data-sheet specification requires an INL of less than ½ LSB, determine the yield of this DAC if everything is ideal except for the random variations in the resistors of the resistor string.

[1] The term "modified Gausian" has been used to describe a distribution that is nearly Gausian. The way this distribution works is as follows. If a sample from the population is taken and the value is within +/-50% of the mean, this sample is accepted. If the sample has a value that is not within this window, it will be discarded and a subsequent sample or subsequent samples will be taken until the value of the sample lies within the +/- 50% window. It is anticipated that in the statistical anaylsis you do in Matlab, there will be no samples that fall outside of this window but just in case it happens, add a counter so that you know how often it occurs. A modified Gausian distribution is used to avoid problems that would occur for extreme values of a random variable which probably occur extremely rarely. For example, without restricting the rare occurrances in the tail of the Guasian distribution, it would be possible for the resistor values to become negative which would not really happen in a physical process.